MThZ 77 (2026) 166–183

Testing Analogies
Between Mathematics and Theism

A Formal Analysis and Evaluation of Silvia Jonas's Argument for Modal-Structural Theism with Paul Bartha's Articulation Model[1]

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Silvia Jonas has argued for interpreting theistic propositions analogically to mathematical propositions, using a modal-structuralist semantics. She claims that objective truth-values for theistic statements can be provided without incurring ontological costs. This paper examines the analogy using Paul Bartha's "Articulation Model" and demonstrates that it can be interpreted as an explanatory-probabilistic analogy with a strong prior association and an adequate potential for generalization, making the analogy functional. There is no internal reason within the analogy to reject the inference, making it reasonable to accept it as sound.

1. Introduction

In religious epistemology, there have been and still are different strategies in understanding religious knowledge and knowledge extension. A very traditional strategy is the method of analogical reasoning. Analogies play a prominent role in natural theology, philosophy of religion and epistemology of religious beliefs in the history of philosophy. Thus, analogical reasoning found its way into Christian theology (i. e. in scholasticism as the term of *analogia entis*), and was often seen as a helpful middle ground between negative theology and univocal theology.[2] The central question almost always was: How can human beings – beyond subjective religious experiences – gain objective knowledge about God when he is

[1] I am very grateful to Franziska Mößner, Elise Tacconi-Garman and Thomas Schärtl for their helpful comments on earlier drafts of this paper. Many thanks to the organizers of the »3rd World Conference of Logic and Religion«, which took place at the *Bahnaras Hindu University* in November 2022 in Varanasi, for inviting me to present core insights of this paper, and to the participants for many helpful comments on my presentation. My presentation was generously supported by the PROSA-LMU-Fund.

[2] For a brief history of analogy from Aristotle to Aquinas see i. e. *Joshua P. Hochschild*, The Semantics of Analogy. Rereading Cajetan's *De Nominum Analogia*, Notre Dame 2010, 4–10. For the role of analogy i. e. in the works of Thomas Aquinas see *Battista Mondin*, The Principle of Analogy in Protestant and Catholic Theology, Den Haag 1963, 85–102; *Roger M. White*, Talking About God. The Concept of Analogy and the Problem of Religious Language, Farnham 2009, 73–102.

associated with a fundamentally transcendent realm inaccessible to the method of deduction?[3] It seems tempting to apply findings from other, more accessible areas, such as, for example, mathematics analogously to religious beliefs, since analogical reasoning is one of the first methods humans develop during adolescence to gain knowledge; and it is a very helpful method in research processes (i. e., in physics, informatics, or biology): Because scenario A has similarities to an unknown scenario B, we – as it seems – are reasonable to conclude that B could be analogous to A and that, therefore, insights can be transferred from A to B.

Analogies between mathematics and theism, which this paper is about, are also well-known in the history of philosophy. Nicholas of Cusa and René Descartes, for example, were convinced that the structure of mathematics permits conclusions regarding the nature of God.[4] Cusas's method of *transumptio*, which approximately can be translated as 'transference,' compares the infinity of a line or the infinite curvature of a circle with the infinite nature of God.[5] Descartes, on the other hand, compares the necessary truth of mathematical propositions (e. g., the sum of angles in a triangle) with the necessary truth of certain theological propositions (e.g. that God exists).[6]

This paper, however, focuses on a very recent and specific analogy between mathematics and theism that Silvia Jonas proposed in 2018.[7] This paper has two main points:

First, it tries to show that the interpretation of theism, as proposed by Silvia Jonas, rests on an analogical conclusion – specifically, that theism can be interpreted in a modal-structuralist way, i. e. analogous to mathematics. The content of this analogy will be analyzed in detail.

Second, the paper demonstrates that Paul Bartha's theory of analogical inference, known as the so-called articulation model, is a suitable tool for examining the plausibility of this analogy.

[3] For the role of analogy in the context of religious language see *White*, Talking About God (see fn. 2), 1–10.

[4] As i. e. Krajewski has shown, there are some fields of Mathematics such as infinite sets or concepts of the number zero where theological metaphors and religious thoughts influenced Mathematicians as well. So a transfer of insights from theism to mathematics is also thinkable, cf. *Stanislaw Krajewski*, Is Mathematics Connected to Religion?, in: Bharath Sriraman (ed.), Handbook of the History and Philosophy of Mathematical Practice, Cham 2024, 2757–2781.

[5] Especially in his works *De docta ignorantia* I,12f. and *De possest*, 43f. Nicholas de Cusa did some research on analogies between mathematics and theism, cf. *Jean Celeyrette*, Mathematics and Theology. The Infinite in Nicholas of Cusa, in: Revue de métaphysique et de morale 70 (2011) 151–165; *Roman Murawski*, Between Theology and Mathematics. Nicholas of Cusa's Philosophy of Mathematics, in: Studies in Logic, Grammar and Rhetoric 44 (2016) 97–110; id., Mathemathics and Theology in the Thought of Nicholas of Cusa, in: Logica Universalis 13 (2019) 477–485; *Jean-Michel Counet*, Mathematics and the Divine in Nicholas of Cusa, in: Teun Koetsier – Luc Bergmans (eds.), Mathematics and the Divine. A Historical Study, Amsterdam 2005, 273–290.

[6] Cf. *Jean Marie-Nicolle*, The Mathematical Analogy in the Proof of God's Existence by Descartes, in: Teun Koetsier – Luc Bergmans (eds.), Mathematics and the Divine. A Historical Study, Amsterdam 2005, 385–403.

[7] Cf. *Silvia Jonas*, Modal Structuralism and Theism, in: Fiona Ellis (ed.), New Models of Religious Understanding, Oxford 2018, 151–170.

2. The Analogy Between Modal-Structuralist Mathematics and Theism

Silvia Jonas has argued in her paper for interpreting theism in much the same way as mathematics can be interpreted, i. e. modal-structuralistically. She points out that God can be understood in the same way as modal structuralists understand numbers, namely as an entity involved in structural relations. Instead of negotiating the ontological status of God and his alleged properties, theists should primarily illuminate the "structural relations [sc. such as the logical, the metaphysical, and the epistemic priority, DB] that define theistic belief."[8] According to Jonas, her interpretation of theism offers objective truth values for theistic statements and a formal logical semantics.[9] Thus, secured theistic statements could withstand non-cognitivist and subjectivist accusations and could also open up possibilities for a conversation with atheists – especially because of their relatively small ontological implications, and because of the formal language.

2.1. The Semantics of modal-structuralist mathematics

The modal-structuralist approach to mathematics interprets mathematical assertions as statements about structures between mathematical entities.[10] According to this approach, which refers back to R. Dedekind's and G. Hellman's considerations regarding the ontological status of mathematical entities, a mathematical statement is true if and only if it is possible that there is a structure that implies the statement or within which it is true.[11] Hellman has porposed the following modal-structuralist semantics for mathematical statements:[12]

For example, an arithmetic statement A like »2 + 2 = 4« is true only if it satisfies the structure of the natural numbers S_N (which in this case are the summary axioms of the so-called Peano arithmetics):

$$\Box\forall S_N\ (S_N \text{ is a structure satisfying the natural-number-requirements} \rightarrow \text{A holds in } S_N)$$

This part, meanwhile, is called the hypothetical component. That it is, on the other side, at least logically possible that such a structure of natural numbers indeed exists is expressed by the so-called categorical component of the MS as follows:

$$\Diamond\exists S_N\ (S_N \text{ is a structure satisfying the natural-number-requirements})$$

[8] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 152 f.
[9] Cf. ibid., 162.
[10] A brief but worthwhile introduction into different kinds of mathematical structuralism and their philosophical implications can be found in *Geoffrey Hellman*, Structuralism, in: Stewart Shapiro (ed.), The Oxford Handbook of Philosophy of Mathematics and Logic, Oxford 2007, 536–562.
[11] For more details about modal-structural mathematics see i. e. *Geoffrey Hellman*, Mathematics without Numbers. Towards a Modal-Structural Interpretation, Oxford 1989; id., Modal-Structural Mathematics, in: Andrew D. Irvine (ed.), Physicalism in Mathematics, Dordrecht 1990, 307–330; Hellman, Structuralism (see fn. 10), 551–560.
[12] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 160. Her semantics is a shortened version of Hellman's modal-structuralist semantics, cf. *Hellman*, Mathematics without Numbers (see fn. 11), 16–33.

Thus, the truth condition for the arithmetic statement »2 + 2 = 4« is, basically, that there may be a structure that logically implies or entails the statement. This eliminative structuralism is, nevertheless, based on the S5-axiom of quantified modal logic while excluding the *Barcan*-formula. The *Barcan*-formula has to be excluded in order to avoid conclusions of the type $\diamond\exists\varphi \rightarrow \exists\diamond\varphi$, which means that it is allowed to derive from the possible existence of φ at the actual existence of a possible φ, which would violate the leading intuitions of modal structuralism.[13] The advantage of choosing the S5-axiom as a framework for modal structuralism is that, in S5, the following is true: $\diamond\exists S_N \rightarrow \Box\diamond\exists S_N$.[14] This means, for instance, that the logical possibility of a structure of natural numbers is itself necessary.

2.2 The Semantics of modal-structuralist theism

In a second step Jonas develops a similar semantics for theistic statements.[15] According to her, theism can be interpreted as a structure that explains how God relates to objects and beings. Jonas identifies three characteristic relations for all kinds of theism. She determines these relations in the logical, metaphysical, and epistemic priority of God. Thus, within a structure God forms, so to speak, the central object G surrounded by the three characterizing relations R_{log}, $R_{metaphy}$, R_{epst} he has to all other objects (like humans, cars, etc.) which are included in the structure. So, according to Jonas, the structure of theism ST can be defined as follows:[16]

1. S_T is a structure containing an infinite number of objects, all of which are in the relation R_{log}, $R_{metaphy}$, R_{epst} to each other.
2. In S_T, there is a central object G, such that G

a. is logically prior to all objects and beings in S_T and
b. is metaphysically prior to all objects and beings in S_T, and
c. is epistemically superior to all objects and beings in S_T.

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[13] Cf. *Hellman*, Structuralism (see fn. 10), 553. There are compelling reasons to exclude the *Barcan*-formula (BF) in general, and particularly for the specific aim within the S5 framework. In general, the BF requires that the domains of all possible worlds are identical: That means that everything that is possible actually and necessarily exists. As a consequence, there would be no contingent objects which is reasonably doubtful (i. e. the fact that Wittgenstein died childless means that there is a possible world in which he had children.). Cf. *Reina Hayaki*, Contingent Objects and the Barcan Formula, in: Erkenntnis 64 (2006) 75–83, 75f. In this specific case, the BF would produce counterproductive consequences, because the point of the modal structuralism is that there is a logical possibility for the existence of a structure SN with no necessity.

[14] Cf. *Hellman*, Mathematics without Numbers (see fn. 11), 17, fn. 8.

[15] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 162–167.

[16] It immediately becomes apparent that what is essentially being discussed are some of the central attributes of perfection in classical theism (omniscience, first cause, etc.) that have been structurally translated. The question is certainly justified as to whether only classical theism is suitable for a structuralist semantics of God-talk and why it is precisely classical theism that has been subjected to such a structuralist interpretation. Jonas does not delve into this further. Even though Jonas emphasizes that essentialist debates about the nature of God are not very promising and that their focus is on the relational proper- ties of God, this nevertheless provokes the charge of onto-theology, as logical, epistemic, and metaphysical superiority are still divine attribute ascriptions. Furthermore, it must be questioned whether the relations proposed by Jonas are truly sufficient for modeling theistic beliefs or whether they are merely necessary. Jonas does not distinctly differentiate in terms of the structuralist translation between necessary and sufficient properties of belief in God, but rather refers to them as "basic relations that arguably […] implicitly define theism," cf. ibid., 165.

You could formally describe the structure S_T as follows:
$S_T := \exists S_T \, \forall x((x \in S_T) \rightarrow (R_{log}(G, x) \vee R_{metaphy}(G, x) \vee R_{epst}(G, x)))$

Any theistic statement (e.g., »God created this world«) could then be understood as a statement about one of the three referenced relations in S_T. One just has to translate the ordinary language statement into structural terms. So the statement »God created this world« means in structural terms that »God is related to the world in logical and metaphysical priority«. From a modal-structuralist point of view, theistic statements always have an implicit structural content which can be explicitly expressed in a structuralist language.

Similar to mathematical statements in mathematical structuralism, two truth-preserving conditions can be stated for theistic statements.[17] Let T be any theistic statement and S_T a structure satisfying theistic requirements. Now, the hypothetical component would be the following:

$\Box \forall S_T$ (S_T is a structure satisfying the theism-requirements $\rightarrow$ T holds in S_T)

This hypothetical component holds that it is necessary for all theistic structures that if this structure satisfies the requirements of theism (a structure such as that defined above), then any theistic statement has a truth-value in S_T. The categorical component, on the other hand, would have to be written down in the following way:

$\Diamond \exists S_T$ (S_T is a structure satisfying the theism-requirements)

The categorical component underlines that it is possible that there is a structure satisfying the requirements of theism. Thus, it is at least logically possible to assume the existence of a structure such as S_T.[18] Therefore, if one wanted to state the truth conditions of any theistic statement, they would have to be delineated in the following way:

A theistic statement is
a) true *iff* it is at least logically possible for there to be a structure satisfying theistic requirements (this means that the term »God« denominates a logically, metaphysically and epistemically superior being etc.) within which the statement is true, or
b) false
i. *iff* it is logically impossible for there to be a structure in which the statement is true or
ii. *iff* it is at least logically possible for there to be a structure satisfying theistic requirements (this means that the term »God« denominates a logically, metaphysically and epistemically superior being, etc.) within the statement is false.

Formulated in terms of possible world semantics, false theistic statements can be false either because there is a possible world in which the statement describes a false state of affairs

[17] Cf. ibid., 166f.
[18] Logical possibility means that "the assumption of there existing a structure [S_T; DB] does not yield a contradiction", cf. ibid., 167.

of fact, or if it is logically impossible that there is a possible world in which the statement describes an obtaining state of affairs of fact.

2.3 Considerations regarding similarities

Jonas's concept of modal-structuralist theism is apparently based on an analogy between mathematics and theism. She, therefore, treats mathematical and theistic statements in the same way and supposes that they have analogical semantics.[19] However, from a higher-order perspective the question arises of how the two semantics of S_N and S_T relate to each other based on their obvious syntactical similarities. Is there some kind of isomorphism between them which would allude to a very strict relation; or is it meant to be a broader structural equivalence? How could we describe this stunning similarity in semantics?

The similarities in the formal semantics can be analyzed as a syntactic or nomic isomorphism between the second-order languages of modal-structuralist mathematics and theism. Hempel characterizes nomic isomorphisms as "syntactic isomorphisms between [the] corresponding sets of laws."[20] So two laws or sets of laws are nomically isomorphic "when [sc. They; DB] share a common mathematical form."[21] The idea undergirding syntactical isomorphism "is always that the equations specifying two rather different models are the same. [...] This does not mean the equations are identical however."[22] As Bartha has shown, the restriction to a common mathematical form is, nevertheless, too strict, because two laws or "statements describing empirical phenomena" can have the same consequences without having the same mathematical form. He argues that two laws are syntactically isomorphic if they imply the same logical consequences or if their formalized principles cause the same consequences.[23] So, we can decipher modal-structuralist mathematics (M_1) and modal-structuralist theism (M_2) as two interpretations or models of the theory of modal structuralism T in a given second-order modal language L. Both interpretations of modal structuralism contain two general principles (the hypothetical and categorical components), sharing the same second-order logical form and underlying principles, namely:[24]

$$\mathrm{Hyp}_{\mathrm{general}} := \Box\forall S_X\ (S_X \text{ is a structure satisfying X-requirements} \rightarrow \text{A holds in } S_X)$$

and

[19] Cf. ibid., 152: "Drawing an analogy between mathematical realism and theism, I offer an account that treats God like modal structuralists treat numbers."

[20] Cf. *Carl Gustav Hempel*, Aspects of Scientific Explanation and Other Essays in the Philosophy of Science, New York 1965, 436.

[21] Cf. *Paul Bartha*, By Parallel Reasoning. The Construction and Evaluation of Analogical Arguments, Oxford 2009, 209.

[22] Cf. *Radin Dardashti – Karim Thebault – Eric Winsberg*, Confirmation via Analogue Simulation. What Dumb Holes Could Tell Us about Gravity, in: British Journal for the Philosophy of Science 68 (2017) 55–89, 66.

[23] Cf. *Bartha*, By Parallel Reasoning (se fn. 21), 209 f.

[24] From a model theoretic point of view, the two principles state that $\Box\forall M_S\ (M \vDash S \rightarrow M_S \vDash \varphi)$ and $\Diamond\exists M_S\ (M \vDash S)$, cf. *Holger Andreas – Georg Schiemer*, Modal Structuralism with Theoretical Terms, in: Erkenntnis 88 (2021) 721–745.

$$Cat_{general} := \Diamond\exists S_X\ (S_X \text{ is a structure satisfying the X-requirements})$$

So, in this case, the two components of modal-structuralist mathematics and theism share a common logical form (the generalized principles as shown above) and have the same consequences (which means that mathematical and theistic propositions are true if there is a structure satisfying certain requirements). The hypothetical component in M_1 and M_2 are instantiations of $Hyp_{general}$ while the categorical component in M_1 and M_2 are instantiations of $Cat_{general}$. Their shared logical form, therefore, can be referred to as syntactically isomorphic.

But another pressing question is why there is a syntactic isomorphism at all? For we can think of many other descriptions of mathematics and theism that are by no means syntactically isomorphic. So, where does this isomorphism arise from in this special interpretation? Now in this regard, the idea of modelling frameworks could be helpful. Its main ideas were introduced by Dardashti et al. to explain how syntactic isomorphisms between a source S and a target T arise.[25] Dardashti et al. state correctly that "we build models using a modelling framework. […] Modelling frameworks almost always involve idealizations, and hence they are usually adequate only under a certain domain of conditions, D. […] In general, adequate means for a particular purpose: accurate to a certain desired degree."[26] Let us, therefore, call M_1 the modelling framework of S under a certain domain of conditions D_{M1}, and M_2 the modelling framework of T under a certain domain of conditions D_{M2}. The source system S is mathematics and the target system T is, of course, theism. The domain of conditions D_{M1} is an abstract structure for mathematics, which is defined by the logical structure within a certain set of numbers (i. e. S_N for the natural numbers satisfying certain requirements such as the axioms of Peano arithmetics). The domain of conditions D_{M2} is an abstract structure for theism (i. e. the defined structures S_T satisfying the requirements as developed by Jonas).[27] So the similarities in the logical form result in the similarities between the domains of conditions. This means that the domains of conditions D_{M1} and D_{M2} are chosen in such a way (and we presuppose that the chosen way is an adequate description of both realms) that the logical structures of M_1 and M_2 define a syntactic isomorphism. Therefore, the isomorphism is a result of the modelling process involving the source and the target. This process starts with a modal-structuralist interpretation of mathematics and theism (D_{M1} and D_{M2}) and results eventually in an isomorphic logical form of its hypothetical and categorical components. But, please note that the simple fact that there is an isomorphism between mathematical and theistic modal structuralism says nothing about the quality or validity of such a modelling procedure. It is a pure description of the formal properties of both and requires further evaluation by justifying the analogy.[28]

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[25] Cf. *Dardashti* et al., Confirmation via Analogue Simulation (see fn. 22), 66–68.
[26] Cf. ibid., 66.
[27] Cf. ibid., 66 f.
[28] Dardashti et al. state correctly that the simple fact of syntactic isomorphism between two realms is not enough to accept analogue behavior. A stable analogy requires additional model-external arguments: "What we need are additional arguments that connect the realisation of the two domains of conditions, and thus imply that the relevant syntactic isomorphism holds between adequate modelling frameworks. […] The most plausible candidates for such arguments would come from additional knowledge […]." Cf. *Radin Dardashti*;*Stephan Hartmann*; *Karim Thebault; Eric Winsberg*, Confirmation via Analogue Simulation. A Bayesian Analysis, 2016

2.4 Possible consequences of the analogy

If the analogy proposed by Jonas is functional and adequate, five points still require further discussions:

1. Formal second-order-semantics of religious language can be offered that provides modal-logically expressible truth values and can be understood interdisciplinarily based on a small number of ontological commitments because it possesses an illuminating model in mathematical structuralism.
2. Theological work and the extension of theological knowledge can be understood as the process of translating religious propositions (either metaphorical or formal) into relational statements about the God-world-relations.
3. This provides a weighty argument for an overly realistic assessment, but not for a naive religious cognitivism.[29]
4. One can justify the project of natural theology by applying mathematical insights on to theism.
5. At the end of the day, the epistemic status of mathematical and theistic statements is – from a formal semantical standpoint – similar.[30]

The question now arises whether this analogical argument is valid and whether truth conditions can be established for this analogy. If such is not the case, or if the analogy does not hold upon further examination, Jonas's proposal may be considered epistemically flawed. But, as suggested previously, Paul Bartha's so-called ʻarticulation model' can be employed to provide sufficient conditions for justifying the inference from modal-structuralist mathematics to modal-structuralist theism, thereby making Jonas's interpretation of theism plausible.

(https://doi.org/10.5282/ubm/epub.28988), 1–19, 5. But it is also important to note that for (in a broader sense hypothetical) analogical arguments (in contrast to analogue simulations and experiments for empirical realms) the supporting evidence cannot count as confirming evidence in a strict sense rather "can establish only the plausibility of a conclusion in the technical sense of justifying the assignment of a non-negligible prior probability assignment", cf. *Radin Dardashti; Stephan Hartmann; Karim Thebault; Eric Winsberg*, Hawking Radiation and Analogue Experiments. A Bayesian Analysis, in: Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 68 (2019) 1–11, esp. 2; see also *Bartha*, By Parallel Reasoning (see fn. 21), § 8.5. However the simple fact that two realms can be modelled and described syntactically isomorphically is a non-negligible prior probability that they behave analogically.

[29] Realistic in this context means epistemologically realistic in respect of the truth-values of mathematical statements, cf. Stewart Shapiro, Structure and Ontology, Oxford 1997, 229. In Shapiros interpretation, Hellman is an antirealist in respect to possible worlds and the ontology of mathematical entities, cf. ibid., 228 f., 232. It is questionable if this analysis is correct for modal-structuralist theism as well, because to be silent or indifferent on ontological questions in theism does not mean to avoid ontological claims personally. It can be argued that theistic propositions can be interpreted modal-structuralistically, with the belief that the logical possibility of a satisfying structure is actualized. Modal structuralism "is a position that is silent on, but fully compatible with, ontological commitment" and "involves no explicit rejection of theistic ontology", therefore cannot be called per se antirealistic, cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 161; 168.

[30] In 2016 Jonas has argued that the epistemological value of mathematical statements can be compared to statements of religious belief on behalf of their rationality standards and the accessibility of their realms, cf. *Silvia Jonas*, On Mathematical and Religious Belief, and on Epistemic Snobbery, in: Philosophy 91 (2016) 69–92.

3. The Articulation Model

3.1 What is an analogy?

First, some conceptual clarification is in order or – to put in bluntly: what is an analogy? An analogy compares two objects – or systems of objects– with respect to relevant similarities. An analogical argument, in turn, is the explicit logical formulation of such a comparison, intended to support a conclusion on the basis of allegedly shared features.[31]

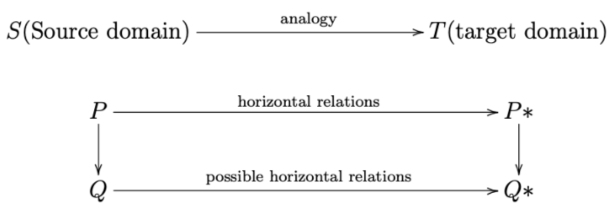

Figure 1: Structure of analogical arguments

3.2. What is an analogical argument?

According to Bartha, an analogical argument has the following form:[32]

1. S [the source domain] is similar to T [the target domain] with respect to property P.
2. S also has the property Q.
3. So T also has the property Q or a property Q* similar to Q. [Conclusion.]

It is important to note that the conclusion of analogous arguments does not necessarily follow from the premises; respectively the premises do not logically imply the conclusion, which is why special semantics are required to arrive at a logically valid conclusion. Furthermore, one can also distinguish positive, negative, neutral and hypothetical analogies:[33]

> *Positive Analogy*: Let P be propositions about S. Suppose the corresponding propositions P* are accepted as valid for T, so that P and P* represent accepted or known similarities between S and T, then P is a positive analogy.
>
> *Negative Analogy*: Let A be propositions that are true in S, and B* be propositions that are true in T. Suppose the analogous propositions A* do not apply in T, and neither do propositions B in S, so that A, ¬A* and B, ¬B* represent accepted or known dissimilarities, then A and B are a negative analogy.
>
> *Neutral Analogy*: An analogy is neutral if it is unclear for the propositions in S whether there are analogous propositions in T.

[31] Cf. *Bartha*, By Parallel Reasoning (see fn. 21), 1 f.
[32] Cf. ibid., 13.
[33] Cf. ibid., 14.

Hypothetical Analogy: A hypothetical analogy is the proposition Q in S in the neutral analogy.

Thus, an analogical inference can be characterized as follows: It is plausible that there is a Q* in T in an analogical way because there are some known similarities in S (positive analogy), despite some known dissimilarities.[34] Mary Hesse has also proposed a helpful terminology. She refers to the semantic relations within a domain as *vertical relations* and to the similarity relation between S and T as *horizontal relations*, as shown schematically in Figure 1 above.[35]

3.3. Types of analogical arguments

Bartha divides analogical arguments into four types on the basis of the logical relations between P and Q (this is the *vertical relation* or *prior association* in Bartha's terminology) (see Figure 2 below).[36] Accordingly, there are analogies that are predictive. That is, if P logically implies a Q in S, then a P* must logically imply a Q* in T as well. This type of analogy can make predictions about a Q* in T based on implication. In explicative analogies, a Q implies a P. That is, if there is a P* and we know that Q implies a P, then in turn, by analogy, there must also be a Q* that implies P*. This type of analogy can be used to explain why a hypothesis is plausible. In functional analogies, the vertical relations run in both directions. Thus, this type of analogy can be reduced to predictive and explicative analogies. In correlative analogies, there is no logical relation between P and Q, only coincidental occurrences. These types of analogies do not allow inferences about a logical or explicative relationship between P and Q. Moreover, relations between P and Q can be deductive or inductive. The relationship between P and Q is, indeed, deductive if P logically implies Q or vice versa. This type of relation is restricted to mathematical theories, whereas inductive relations can occur in all four types.

According to Bartha, the further up and to the left in the table an argument is, the stronger it is. Bartha correctly points out that this first characterization does not yet say anything about the formal semantics of analogical arguments, which have to be stated separately.

3.4. The Articulation Model

A prerequisite for a good and plausible analogy, according to Bartha, is "a clear connection, in the source domain [sc. that are the horizontal relations; D.B.], between the known similarities (the positive analogy) and the further similarity that is projected to hold in the target domain (the hypothetical analogy). I call this the prior association."[37]

Accordingly, Bartha's basic theory of analogical arguments – his so-called Articulation Model – is based on two principles:

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[34] Cf. ibid., 15

[35] Cf. *Mary Hesse*, Models and Analogies in Science, Notre Dame 1963, 59.

[36] Cf. *Bartha*, By Parallel Reasoning (see fn. 21), 95–97.

[37] Cf. ibid., 94.

Prior association: In S, there must be a clear connection between the known similarities (positive analogy) and the similarity transferred to T (hypothetical analogy).[38]

Potential of generalization: A good analogical argument provides at least no reasons not to transfer the prior association in S to T. Put positively: It must be plausible to assume the same kind of prior association in T as in S.[39]

The principle of prior association states that plausible analogical arguments do not merely arbitrarily transfer similarities from S to T, but apply an explicable, vertical relation within S to T by analogy. This relation can be logical, causal, explanatory, or correlative (see Figure 2).[40] The principle of potential generalizability expresses that the analogical relation must be transferable to T in the first place.

The advantages of Bartha's theory in comparison to other semantics of analogical reasoning can be identified as threefold.[41] First, Bartha qualifies analogical inferences based on their direction of inference (Q to P or vice versa, etc.), i.e., whether the inference is deductive or inductive. Second, he gives criteria for which type of analogy, which factors of the analogy are functionally relevant or critical and which are not, since not always all factors of an analogy have a direct influence on its functionality. And third, Bartha ranks analogies according to the strength of their inference (mathematical-predictive analogies are to be rated stronger than correlative ones).

DIRECTION OF PRIOR ASSOCIATION

MODE	**Predictive** (From P to Q)	**Explanatory** (From Q to P)	**Functional** (Both directions)	**Correlative** (No direction)
Deductive	Mathematical	Abductive	—	—
Inductive	Predictive/ Probabilistic	Abductive/ Probabilistic	Functional	Correlative

Figure 2: Types of analogical arguments sorted by the type of prior association, cf. Bartha, By Parallel Reasoning, 98.

4. Testing the Analogy with Bartha's Articulation model

After this brief introduction to Bartha's articulation model, Jonas's analogous argument, on which the concept of modal-structuralist theism is based, can be tested for its plausibility

[38] Cf. ibid., 25, 94.
[39] Cf. ibid., 25.
[40] Cf. ibid., 98.
[41] Another well-known semantics for analogical reasoning is i. e. the structure-mapping-semantics developed by Gentner, cf. i. e. *Dedre Gentner*, Structure-mapping. A Theoretical Framework, in: Cognitive Science 7 (1983) 155–170.

thanks to Bartha's proposal. First, it is necessary to clarify what type Jonas's argument belongs to according to Bartha's classification, i. e., whether it proceeds basically deductively or inductively and whether it is of the predictive, explanatory, functional, or correlative type.

The assumption that we may have encountered a correlative type can be excluded for Jonas's argument, since the modal structuralism of the natural numbers developed in S is not based merely on coincidence or statistical correlation but has an explicable connection to the other features in S. Jonas's argument can be classified as an inductive argument according to Bartha's definition of deduction and induction, because neither P logically implies Q nor vice versa. Additionally, Jonas notes that the categorial component (here Q) "must be derived from uncontroversial facts," meaning it must be inductively inferred from empirical evidence.[42] Thus, the analogous argument can be neither mathematically-predictive nor explanatorily-abductive. But it cannot be predictive-probabilistic either, because in this type of argument, the prior association is a causal explanation.[43]

After excluding all unsuitable possibilities, only the explanatory-probabilistic type remains. This argument type explains a phenomenon E (e. g., mathematical entities) by Q ("part of a causal explanation"), additional hypotheses C, background knowledge B and positively valent relevant factors contributing to Q called φ+, despite relevant factors speaking against Q called φ– where Q ⊆ φ+.[44] Note that the direction of inference of the abductive-probabilistic type runs from Q to P (Q → P).[45]

4.1. Formal analysis of the argument

Source Domain [S] (Natural numbers)	**Target Domain [T] (Theism)**	**Explanation**
ϕ^1	ϕ^1*	[N] and [T]
ϕ^2	ϕ^2*	$[S^N]$ and $[S^T]$
ϕ^3	ϕ^3*	[S5-Axiom]
ϕ^4	ϕ^4*	[hypothetical component of modal structuralism]
Q	Q^*	[categorical component of modal structuralism]

- Potentially relevant factors: F := {φ, Ψ, B}
- Relevant factors explicitly given in the argument: φ := {ϕ^1, ϕ^2, ϕ^3, ϕ^4, Q}
- Factors given in other important arguments for the same or competing conclusions: Ψ := { } → empty, because there is still just one known argument for the conclusion
- Implicit background factors or unstated assumptions: B
- Phenomenon to be explained: E := {natural numbers, theism}
- Positive relevant factors: φ+ := {ϕ^1, ϕ^2, ϕ^3, ϕ^4, Q}
- Additional hypothesis: C := {ϕ^4}

[42] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 160.
[43] Cf. *Bartha*, By Parallel Reasoning (see fn. 21), 113.
[44] Cf. ibid., 129.
[45] Cf. ibid., 98.

- Explicit defeating factors: $\varphi- := \{\}$
- Critical factors: $\varphi^C := \{\phi^1, \phi^2, \phi^3, \phi^4, Q\}$

4.2. Prior association

Thus, according to Bartha, the prior association of the abductive-probabilistic argument type is as follows:[46]

> Some phenomenon E because $\varphi+$ and C and Π^- [absence of Π] despite $\varphi-$, where $Q \subseteq \varphi+$.

In other words, a random phenomenon E is to be explained by an explanans Q. There are some additional assumptions C that are needed to explain E. Furthermore, "the positive analogy P typically overlaps significantly with the set E of observable consequences and may extend other elements of the explanans."[47]

When applied to this argument, these factors can be assigned as follows: E is the phenomenon of the natural numbers. $\varphi+$ are the explicit, positively valent factors ϕ^1, ϕ^2, ϕ^3, Q – these are the set of natural numbers, a (possibly existing) structure satisfying the natural-number-requirements and the S5-axiom.

C is the additional hypothesis ϕ^4 – the hypothetical component of modal structuralism, which serves as a structuralist "translation pattern" (as Jonas says)[48] for statements about natural numbers and was newly introduced by Hellman. Π^- (which is the absence of "defeaters" of $\varphi+$ and C) is given because, for the present argument in S, there are no factors given that could corrupt $\varphi+$. $\varphi-$ is empty in the present case because there are no explicit factors in S that overtly argue against Q.[49]

According to Bartha, this argument type has the following two plausibility preconditions:[50]

1. No defeating condition for any contributing cause in the explanation may be known to hold in the source domain.
2. The additional assumptions C must be justified.

The first point is satisfied because there are no factors Π in S speaking against $\varphi+$ and C. The second requirement is also satisfied because it is justified for a theory of mathematical structuralism to state formal semantics such as that of ϕ^4.[51]

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46 Cf. ibid., 129.
47 Cf. ibid., 129.
48 Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 160.
49 "A defeating condition is a circumstance that, if it were to obtain, would convert a contributing cause to a counteracting cause (or vice versa)." Bartha notes that aleatory explanations "need not mention every contributing cause. Nor need they include, for each contributing cause mentioned, statements about all known defeating conditions. Completeness just means that for each contributing cause that is included, no defeating condition is known to obtain in the source domain", cf. *Bartha*, By Parallel Reasoning (see fn. 21), 115.
50 Cf. ibid., 129.
51 Here we can refer to Hellman's remarks and semantics of mathematical structuralism, cf. *Hellman*, Structuralism (see fn. 10), 551–560.

4.3. Relevant factors

According to Bartha, for abductive-probabilistic arguments, all factors (except for factors counteracting the analogy called $\varphi-$) in the prior association are critical and therefore relevant. Moreover, it is important to note that the effects of Q (called the "scope(Q)") and causes of Q are also considered critical. Effects of Q are possible consequences of Q, along with the additional assumptions in the prior association. Critical causes of Q are thwarting factors Π, as are additional assumptions C, where "their analogs are typically formulated in speculative terms."[52] Accordingly, the analogy is corrupted if any of these factors belong to a negative analogy.[53] That means that there would be no corresponding element in T.

In this example, all factors are critical because ϕ^1, ..., ϕ^4, and Q are essential for the prior association. That is, if one of these critical factors is omitted, the analogy is not plausible. Since none of the factors in S belong to a negative analogy either, the analogy is unchallenged up to this point.

4.4. Potential of generalization

Now let's take a look at the second principle, the potential of generalization. Bartha defines the prima facie plausibility criteria for abductive-probabilistic arguments as follows:[54]

1. There must be an intersection of E and the positive analogy.
2. There must be a difference between the critical factors and negative analogies.
a. Effects of Q [called the "scope(Q)"] must not constitute a negative analogy.
b. There must be no known corrupting factors (Π^*) in T. C and $\varphi+$ must not belong to a negative analogy.

The first requirement is fulfilled because between E (phenomenon of natural numbers) and the positive analogy (that is the proposition that natural numbers and theistic statements can be interpreted structuralistically), structuralism is the intersection. Number 2a is satisfied because the effects and consequences of Q (which is a modal-structuralist interpretation of mathematics) are not a negative analogy to theism. After all, the latter can also be formulated in modal-structuralist terms (as Jonas has shown). C and $\varphi+$ both belong to a positive analogy and are therefore indisputable. Also, there are no corrupting factors (called Q*) in T. Thus, condition 2b is also fulfilled; and the analogy is prima facie plausible. Let us now also apply three other qualitative plausibility criteria to Jonas's argument: These are the strength of the prior association, the extensibility of positive analogies and other analogies justifying the argument.

The argument reviewed in this paper exhibits quite a strong prior association, because the number of positively valent factors $\varphi+$ is high, while there are no (obviously) corrupting factors $\varphi-$.

The number of positive analogies between S and T can also be extended, i. e., by considering the similarities between mathematical and theistic epistemology (both acquire

[52] Cf. *Bartha*, By Parallel Reasoning (see fn. 21), 132
[53] Cf. ibid., 130.
[54] Cf. ibid., 132.

knowledge by perceiving objects and constellations of objects that are connected with each other by logical or causal relations, by abstracting from these objects to general structural relations and by deriving the "logical possibility of there being a constellation exemplifying this structure"[55]). We can also imagine positive analogies between the background knowledge of the two domains, i. e., the ontological status of the theistic and mathematical existence quantor are both under consideration or unclear.

The criterion of multiple analogies is not fulfilled yet, because so far Jonas is the only scholar who compares modal structuralism in mathematics with theism.

All in all, the qualitative plausibility of the analogical argument is positive, especially because Jonas is able to show strong prior associations both in S and T while circumventing corrupting factors (especially ontological commitments) by its modal character.

5. Conclusion and Critique

The paper tried to demonstrate several key points: Silvia Jonas's analogy can be adequately represented within the formal language of Bartha's *Articulation Model*. The analysis of the analogical argument has identified the critical factors and prior associations involved, categorizing Jonas's argument as an abductive-probabilistic type, which is considered a relatively strong form of analogical reasoning. Additionally, the articulation model has proven to be suitable for analyzing analogies between mathematics and theism, making it a valuable tool in this area of philosophy. Notably, this appears to be the first application of Bartha's approach in the philosophy of religion. Finally, the analogy between modal-structuralist mathematics and theism is plausible, thereby justifying such an interpretation of theism.

To anticipate critique, it must be said, considered also as a closing remark, that the premise of Jonas's argument that mathematical statements could be and should be interpreted modal structuralistically and only in this way must be accepted for the sake of the argument. But the modal structuralist interpretation of mathematics is neither a matter of fact nor uncontroversial but an open and highly disputed question within the philosophy of mathematics.[56] Therefore, the paper wanted to demonstrate primarily that Bartha's articulation model provides a detailed hermeneutics for mathematical-theistic analogies, proving effective in assessing analogies between rather distant areas. Even if one does not agree with theistic ontological premises or background assumptions, the analogy offers a way to formalize and assign truth values to a theistic semantics.

Upon closer examination, it becomes clear that the analogy presented here, and evaluated using formal methods, makes several significant philosophical and theological assumptions

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[55] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 169 f.

[56] Even modal structuralism is just one possibility to model mathematical structuralism, cf. *Hellman*, Structuralism (see fn. 10), 537 f.; *Erich Reck*; *Michael Price*, Structures and Structuralism in Contemporary Philosophy and Mathematics, in: Synthese 125 (2000) 341–383. For a critique of modal structuralism cf. *Shapiro*, Philosophy of Mathematics (see fn. 29), 228 f.

that are neither trivial nor uncontroversial. Three of these implicit premises need to be briefly addressed in the following paragraphs.

Silvia Jonas' interpretation of belief in God, specifically the modal-structuralist reformulated cognitive aspect discussed here, can be classified as a theoretical undertaking that aims to explicate its semantic structure in analogy to mathematical structuralism. This approach is based on a structuralist theory of religious statements, for which Lorenz Bruno Puntel can be regarded as an influential figure in the philosophy of religion and the philosophy of science.[57] Puntel's elaborated structuralist theory framework, which he developed primarily for philosophy and which has also been applied to theological theory building[58], seeks to distinguish itself from other conceptions in the philosophy of science, such as the logical-empiricist Syntactic view or the still influential Semantic view of Theories.[59] This distinction largely stems from the weaknesses of these theoretical frameworks – such as, in the first case, their fixation on observational sentences and their rigid axiomatic translation, and in the second case, from metamathematical doubts about the applicability of set-theoretical categories to the model-theory relationship. In this context, the concept of structure seems to offer an elegant and promising solution. However, the concept of structure, as Shapiro highlights in his overview article on mathematical structuralism, has an ontological fault line, which, when analogously applied to theology, reopens the "old wounds" of the onto-theology critique.[60] If one does not want to completely exclude or leave unanswered the question of the ontological status of structures, and if the concept of structure is analogously and modally softened when applied to God-talk, then – despite Silvia Jonas' recommendation to set aside ontological debates about the properties and essence of God – the concept of structure merely shifts the question to another area: What are the ontological commitments of a modal-structuralist theism, and ultimately, what kind of entities are structures, and what is the relationship between God and his structurally described properties to the properties of these structures? In other words, by adopting a modal-structuralist theism analogously adapted from mathematics, one *nolens volens* invites the charge of onto-theology, because despite all assurances to the contrary, the status of structures and the theological entities described by them is not off the table, even when modal structuralism is embraced. These remarks alone, of course, do not yet determine whether such a charge is valid or not, but it certainly gives modal-structuralist theism further ontological homework to do.

Secondly, Silvia Jonas' proposed model aims to rationally explicate and secure belief in God, analogous to the structuralist foundation of mathematical entities, against a practical-ethical or psychological reduction of religious convictions. This rational explicating and

[57] Cf. *Lorenz Puntel*, Struktur und Sein. Ein Theorierahmen für eine systematische Philosophie, Tübingen 2006.
[58] See, i. e., the theory concepts influenced by Puntel from *Christina Schneider*, »Gott« als theoretischer Term, in: Benedikt Paul Göcke (ed.), Die Wissenschaftlichkeit der Theologie. Band 1: Historische und systematische Perspektiven, Münster 2017, 315–336; *Ruben Schneider*, Struktural-systematische Theologie. Theologie als Wissenschaft vor dem Hintergrund der transzendentalen Wende, in: Benedikt Paul Göcke (ed.), Die Wissenschaftlichkeit der Theologie. Band 1: Historische und systematische Perspektiven, Münster 2017, 33–83.
[59] Cf. *Puntel*, Struktur und Sein (see fn. 57), 162–188.
[60] Cf. *Hellman*, Structuralism (see fn. 10), 537 f.

securing of the semantics of religious discourse is achieved through modal structuralism. The truth value of religious statements is determined by the possible existence of truth-making structures. Thus, the model aligns itself with a realist and cognitivist tradition of interpreting religious God-talk. Furthermore, an epistemically optimistic attitude is also arguably evident. But both, the realism implied by the conception of truth value and the cognitivist interpretation, are undoubtedly open to criticism. The most compelling objection seems to be that the truth values for the semantics of religious discourse in the modal-structuralist conception can only be securely determined thanks to a kind of meta-perspective or thanks to revelation. How do religious speakers know whether the truth-making structures of their God-talk exist or which statements within them have the kind of truth value in question? To justify the truth value or the truth-making structure, rational reasons must inevitably be provided, whose plausibility is in turn based on epistemological premises. In mathematics, for example, structuralist descriptions can be justified set-theoretically, but how can a structuralist semantics be theologically justified evantually? A practical-ethical justification of the semantics of religious discourse, as proposed by Saskia Wendel for instance, seems conceivable and worth considering, especially since Silvia Jonas emphasizes the focus on the individual and personal relationships of believers to God rather than on questions about the divine essence.[61]

Thirdly, despite all criticism, the model character of the analogy needs to be taken into consideration. Models are deeply purpose-dependent and thus assumption-laden scientific tools, whose purpose and goal-orientation are essential for an adequate understanding.[62] The plausibility of the analogy, and thus of the accompanying model of religious discourse, also depends on the purpose and goal-orientation of the model and the effectiveness of the implicit assumptions, such as mathematical structuralism, a realist understanding of language, and an epistemically optimistic attitude toward theism. The analogical theoretical investigations conducted here cannot, of course, justify these premises, as they focus on the analogy itself rather than the justification of the premises, but can serve as an abductive type of making plausible and justifying possibilistically the rationality of theistic semantics. Nevertheless, the endeavor to conceive of a structuralist "universal semantics" of religious discourse, which explicitly claims a theistic perspective and borrows semantic concepts from mathematics, appears both fascinating and problematic. This approach is driven by the ethical demand in science that, before addressing metaphysical and ontological questions in the philosophy of religion, a meaningful semantics of religious statements must be established. Therefore, while belief in God doesn't necessarily need to be viewed as a theoretical endeavor, engaging in a responsible reflection on religious statements still requires providing a clear and understandable semantics for those statements.[63] The plausibility of

[61] Cf. *Jonas*, Modal Structuralism and Theism (see fn. 7), 165.
[62] The context and purpose-bound nature of models, along with their adequacy criteria, was highlighted i. e. by *Wendy Parker*, Model Evaluation. An Adequacy-for-Purpose View, in: Philosophy of Science 87 (2020) 457–477.
[63] See also Pannenberg's call for the modeling of theological theories for the purpose of plausibility: "Theological statements, like other scientific propositions, are situated within theoretical contexts and can only be evaluated in terms of their function within these theoretical frameworks. This leads to the ethical demand for theology to explicitly and systematically develop theoretical models. These models and the statements associated with them

a religious semantics in analogy to the semantics of mathematical statements at least supports the hypothesis of the rational and interdisciplinary compatibility of belief in God. Silvia Jonas's proposal of a modal-structuralist interpretation of theism could thus be described, if not as a comprehensive world model, then most appropriately as a linguistic model that attempts to semantically model the objects and subjects of religious discourse, while remaining, of course, fallible and limited. In this sense, it is fitting to conclude here with the well-known quip of the famous statistician and modeler George Box: "All models are wrong, but some are useful."[64]

Silvia Jonas hat argumentiert, theistische Aussagen analog zu mathematischen Aussagen unter Verwendung einer modal-strukturalistischen Semantik zu interpretieren. Sie behauptet, dass objektive Wahrheitswerte für theistische Aussagen bereitgestellt werden können, ohne ontologische Kosten zu verursachen. Diese Arbeit untersucht die Analogie anhand von Paul Barthas ‚Articulation Model' und zeigt, dass sie als explanatorisch-probabilistische Analogie mit starker vorheriger Assoziation und ausreichendem Potenzial für Verallgemeinerung interpretiert werden kann, wodurch die Analogie funktional ist. Es gibt keinen internen Grund innerhalb der Analogie, die Schlussfolgerung abzulehnen, sodass es vernünftig ist, sie als sinnvoll zu akzeptieren.

then take the form of hypotheses," cf. *Wolfhart Pannenberg*, Wissenschaftstheorie und Theologie, Frankfurt a. M. 1973, 335 [translation; DB].

[64] Cf. *George Box; Norman Draper*, Empirical Model-Building and Response Surface, New York 1986, 424.